

Maths Progressions - Number and algebra

This document was created by Clevedon School staff using the NZC, Maths Standards and Numeracy Framework, with support from Cognition Education consultants. It is indicative of the maths knowledge and strategies required to meet the curriculum and national standards at different year levels. A differentiated programme will be needed to ensure all students are scaffolded to progress in their learning. The learning intentions listed in this document are not in linear order and are not intended to be used as such. The needs of the learners, as identified through formal and informal data gathering, drives learning at all times at Clevedon School. The maths progressions for [geometry, measurement](#) and [statistics](#) should be used in conjunction with these progressions.

At Clevedon School maths programmes are characterised by the following:

- A problem solving approach, rather than a 'number crunching' one
- Differentiated practices, based on needs, as ascertained by current and reliable data
- Integration of the various strands of maths as much as possible and integration of the numeracy domains
- Integration of maths in other subjects and in real life settings
- Teachers and students using the correct vocabulary as set out in this document to ensure students experience the same language of learning from class to class
- The development of assessment capable learners who understand and can articulate the following:
 - Where am I going? What are my goals?
 - How am I going? What progress is being made towards the goal?
 - Where to next? What activities need to be undertaken next to ensure progress?
- [Effective pedagogy in mathematics](#) - The International Academy of Education's research on improving student achievement in mathematics, based on Best Evidence Synthesis is used to inform classroom practice. This [short guide](#) with questions to consider when reflecting on classroom practice may be helpful.
- A strategic approach is used for teaching 'mathematical behaviours' as set out in this document. Learning intentions for mathematical behaviours must be a part of each lesson.
- Classroom discourse on mathematics is strategically developed. Use this [document](#) to support the establishment of a discourse based classroom.

How to use this document:

- These progressions are not a checklist to be worked through. They can form the basis of the maths programme and should be used to inform planning. Students may be working at multiple stages in different areas of their learning.
- Data analysis, identifying of goals and any other planning will start with the maths progressions but other resources can also be drawn on.
- Progressions can be used to plan and teach a specific group and sometimes for 'clinics' i.e. students across the class with the same identified learning need.
- Share the progressions with students (age appropriate), so they know where they are, where they are going and what their next learning step is.
- All lessons must include learning intentions from the 'mathematical behaviours' section. These should be specifically planned for, taught and monitored.

Other relevant school based documents:

- [Maths assessment timeline and guidelines](#)
- [Assessment capable learners at Clevedon School](#)
- [Literacy and numeracy achievement map](#)
- Talk moves [version 1](#), [version 2](#)

Other relevant documents:

- [The New Zealand Curriculum - mathematics and statistics achievement objectives](#)
- [The Mathematics Standards](#)
- [Best Evidence Synthesis - Mathematics](#)

Maths Progressions - Number and algebra

Stage 0

Emergent

Early level 1

During these school years, number should be the focus of 60-80% of mathematics teaching time.

Mathematical Behaviours (what mathematicians do)



I am learning to...

| | |
|-------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| Explain my mathematical thinking | Explain others' strategies by repeating or re-voicing what they have said |
| Use the most efficient strategy to solve a problem e.g. I know that counting on from the largest number is more efficient than counting from 1. | Agree or disagree with someone else's strategy or answer, and explain why I think this |
| Listen actively | Say what I am good at in maths and what I need to work on |
| Identify maths in the world around me | Use mathematical vocabulary |
| Pose problems and conduct mathematical investigations | |

Knowledge

I am learning to...

Number Identification, Sequencing and Ordering

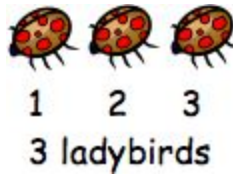
| | |
|-----------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Read numbers to 5 first and then to 10 | 0, 1, 2, 3, 4, 5 6, 7, 8, 9, 10 |
| Count forwards to 5 first and then to 10 | 0, 1, 2, 3, 4, 5...10 |
| Count backwards from 5 first and then from 10 | 5, 4, 3, 2, 1, 0 |
| Say the number after a number (in the range 1- 5) |  3, 4, ____ |
| Say the number before a number (in the range 1 – 5) |  ____, 4, 5 |
| Order numbers to 5 first and then to 10 | 5 3 1 2 4 |

Strategy

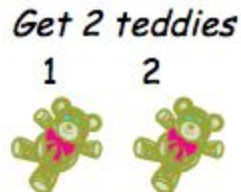
I am learning to...

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Count a set of objects up to 5 first and then to 10 by one-to-one matching



Form a set of objects up to 5 first and then to 10 by one-to-one matching



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Stage 1

After 20 weeks at school

One-to-one counting

During these school years, number should be the focus of 60-80% of mathematics teaching time.

Mathematical Behaviours (what mathematicians do)

I am learning to...

Explain my mathematical thinking

Explain others' strategies by repeating or re-voicing what they have said

Use the most efficient strategy to solve a problem e.g. I know that counting on from the largest number is more efficient than counting from 1.

Agree or disagree with someone else's strategy or answer, and explain why I think this

Listen actively

Say what I am good at in maths and what I need to work on

Identify maths in the world around me

Use mathematical vocabulary

Pose problems and conduct mathematical investigations

Knowledge

I am learning to...

Number identification, Sequencing and ordering

Read numbers to 10

7, 6, 8, 5


Count forwards to 10

0, 1, 2, 3, 4, 5...10


Count backwards from 10

10, 9, 8, 7, 6, 5...

Say the number after a number (in the range 1- 10)


4, 5, ____

Say the number before a number (in the range 1 – 10)

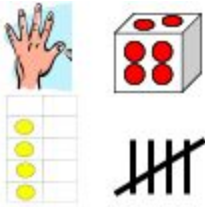

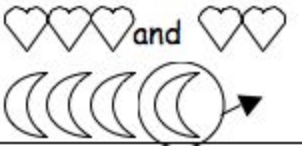

____, 4, 5

Order numbers to 10

5 3 1 8 2 7

Grouping

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| | |
|----------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| Instantly recognise patterns to 5 |  |
| <div>Strategy</div> <div>I am learning to...</div> | |
| Count a set of objects up to 5 first and then to 10 by one-to-one matching |  |
| Form a set of objects up to 5 first and then to 10 by one-to-one matching |  |

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Stage 2/3 Counting from one

After 1 year at school

Early level 1

During these school years, number should be the focus of 60-80% of mathematics teaching time.

Mathematical Behaviours (what mathematicians do)

I am learning to...

Explain my mathematical thinking

Explain others' strategies by repeating or re-voicing what they have said

Use the most efficient strategy to solve a problem e.g. I know that counting on from the largest number is more efficient than counting from 1

Agree or disagree with someone else's strategy or answer, and explain why I think this

Listen actively

Say what I am good at in maths and what I need to work on

Identify maths in the world around me

Use mathematical vocabulary

Pose problems and conduct mathematical investigations

Knowledge

I am learning to...

Number identification

Read any number to 20

17, 16, 18, 15

Say the 'ty' numbers

Twenty, forty

Sequencing and ordering


Count forwards from any number up to 20

14, 15, 16...


Count backwards from any number up to 20

15, 14, 13...

Say the number after a number (in the range 1- 20)


14, 15, ____


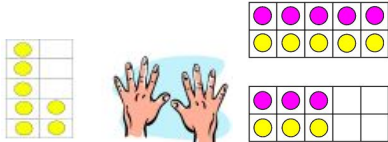

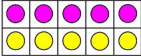
Say the number before a number (in the range 1 - 20)


____, 14, 15

Order numbers to 20

5 3 1 18 20 7

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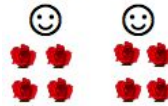
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|---------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Skip count in 2s, 5s and 10s | 2, 4, 6, 8...20 5, 10, 15, 20... 10, 20, 30.... |
| Grouping, place value and basic facts | |
| Know groupings within 5 Know groupings within 10 |  5 and 2 7 and 3 |
| Know doubles to 10 | $5 + 5 = 10$ |
| Know the place value for 'teen' and 'ty' numbers up to 20 | $13 = 10 + 1$ (bundle 1 ten and 3 ones) |
| Instantly recognise patterns to 10 (doubles and 5 based) |  |
| Fractions | |
| Read symbols for halves and quarters | $\frac{1}{4}$ $\frac{1}{2}$ |
| Strategy | |
| I am learning to... | |
| Addition Solve simple addition problems by counting all the objects with materials or in my head | $6 + 3 =$  |
| Subtraction Solve simple subtraction problems by counting all objects with materials or in my head | $6 - 3 =$  |
| Multiplication Solve simple multiplication problems by counting all the objects | There are 4 lolly jars, each lolly jar has 2 lollies in it. How many lollies are there altogether? Counting all (one by one) is ok. If a child skip counts they exceed the expectations |

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Fractions

Find $\frac{1}{2}$ and $\frac{1}{4}$ of shapes or sets to 20 by equal sharing of the objects

$$\frac{1}{2} \text{ of } 8 =$$



Equations and Expressions

I am learning to...

Communicate and explain my strategy for counting and grouping using words, numbers and pictures

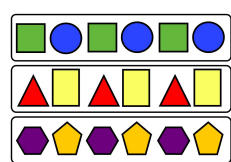


I counted all the balls to get 8.

Pattern and Relationships

I am learning to...

Continue sequential patterns



Identify what the unit of repeat is in a pattern

Identify unit of repeat e.g. green square, blue circle.

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Stage 4 Advanced counting

After 2 years at school

At level 1

During these school years, number should be the focus of 60-80% of mathematics teaching time.

Mathematical Behaviours (what mathematicians do)



I am learning to...

| | |
|------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| Explain my mathematical thinking | Explain others' strategies by repeating or re-voicing what they have said |
| Use the most efficient strategy to solve a problem e.g. I know that counting on from the largest number is more efficient than counting from 1 | Agree or disagree with someone else's strategy or answer, and explain why I think this |
| Listen actively | Say what I am good at in maths and what I need to work on |
| Identify maths in the world around me | Use mathematical vocabulary |
| Pose problems and conduct mathematical investigations | |



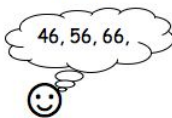
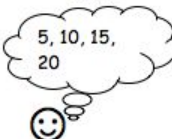
Knowledge

I am learning to...

Number identification, sequencing and ordering

| | |
|-------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| Read any number to 100 | 17, 16, 58, 65, 99 |
| Count forwards from any number up to 100 | 34, 35, 36... |
| Count backwards from any number up to 100 | 35, 34, 33... |
| Say the number after a number in the range 1-100 |  54, 55, ____ |
| Say the number before a number in the range 1-100 |  ____, 54, 55 |
| Order numbers to 100 | 25 33 12 18 20 77 |
| Count forwards and backwards in 2s, 5s and 10s to 100 | 2, 4, 6, 8, ..., ..., 100 5, 10, 15, ..., ..., 100 10, 20, 30, ..., ..., 100 |
| Know number of 10s in decades | There are 6 tens in 60 |

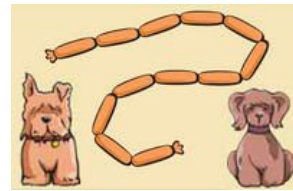
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| Grouping and basic facts | |
|------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|
| Know groupings within 20 | $17 + 3$ $4 + 16$ |
| Know teen number facts | $10 + 6 = 16$ $10 + ? = 18$ |
| Know multiples of 10 that add to 100 | $30 + 70 = 100$ |
| Know doubles and halves to 20 | $3 + 3$ $6 - 3$ $\frac{1}{2}$ of 6 double 3 |
| Know addition facts to 10 | $4 + 3 = 7$ |
| Fractions | |
| Read unit fractions | $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{10}$ |
| Strategy | |
| I am learning to... | |
| Addition Solve addition problems by counting on from the largest number in my head | $16 + 3 =$  |
| Subtraction Solve subtraction problems by counting back from the largest number in my head | $32 - 3 =$  |
| Solve addition and subtraction problems by counting on or back in ones and tens | $36 + 40 =$  |
| Multiplication Solve multiplication problems by skip counting in 2s, 5s or 10s | $4 \times 5 =$  |

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Multiplication/division/fractions

Solve multiplication and division problems by using knowledge of doubles and halves

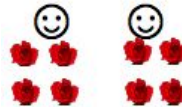


12 sausages shared between 2 dogs. Double 6 is 12.

Fractions

Find $\frac{1}{2}$ and $\frac{1}{4}$ of sets of shapes, sets and numbers by equal sharing

$\frac{1}{2}$ of 8 =

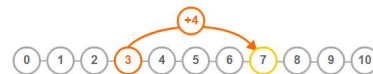
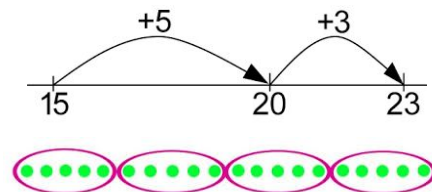


Students must be able to understand the concept of equal sharing with shapes, sets and numbers - not just shapes

Equations and Expressions

I am learning to...

Communicate and explain my strategy for counting, grouping and equal sharing using words, numbers and pictures



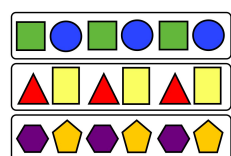
$$3 + 4 = \square$$

I began counting from the smallest number 3 and counted on 4 to get 7.

Pattern and Relationships

I am learning to...

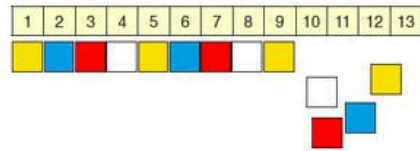
Create and continue sequential patterns



Identify what the unit of repeat is in a pattern

Identify unit of repeat e.g. yellow square, blue square, red square, white square...

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What would the 13th shape be in this pattern? The expectation is that the student continues the pattern one cube at a time until they identify a yellow cube at 13.


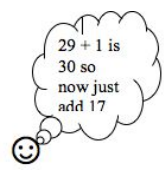
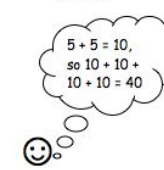
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Stage 5 Early additive part-whole In year 3 and 4 Year 3-E level 2/ Year 4-At level 2

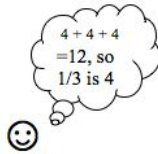
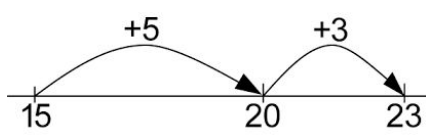
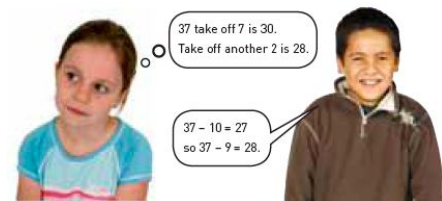

During these school years, number should be the focus of 60-80% of mathematics teaching time.

| Mathematical Behaviours (what mathematicians do) | |
|----------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| I am learning to... | |
| Explain my mathematical thinking orally, visually or in writing | Engage in mathematical discussions with others |
| Explain others' strategies by repeating or re-voicing what they have said | Use the most efficient strategy to solve a problem e.g. I know that for this problem using a place value strategy is more efficient than counting on. |
| Agree or disagree with someone else's strategy or answer, and explain why I think this | Listen actively |
| I know what I am good at in maths, what my gaps are and my next steps | Use mathematical vocabulary |
| Pose problems and conduct mathematical investigations | Make connections with what I am learning in maths to other curriculum areas and the world around me |
| Knowledge | |
| I am learning to... | |
| Number identification, sequencing and ordering | |
| Read any number to 1000 | 170, 316, 508, 685, 990 |
| Count forwards and backwards by 1s, 10s, 100s | 1000...350, 345, 340 |
| Order numbers to 1000 | 126, 433, 754 |
| Say the number 1 more, 10 more, 100 more |  145, 155, ____ |
| Say the number 1 less, 10 less, 100 less |  ____, 154, 165 |
| Skip count forwards and backwards in 3s (as well as 2s, 5s and 10s) | 3, 6, 9, 12..., 30 |
| Grouping and place value | |
| Know groupings of 1s, 10s and 100s to at least 1000 | 327 = 32 tens 924 = 9 hundreds or 92 tens |

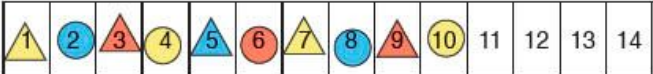
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|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Know groupings to 100 | 43 and 57 I know that 3 and 7 make 10 and that 40 and 50 make 90 so that's 100. |
| Round 3 digit numbers to the nearest 10 or 100 | 246 → 250 |
| Basic facts | |
| Know addition subtraction facts to 20 (instant recall) | $12 + 8 = 20$ $20 - 5 = 15$ |
| Know multiples of 100 that add to 1000 (instant recall) | 400 and 600 |
| Know multiplication & division facts for x2, x5, x10 (instant recall) | $5 \times 2 = 10$ $60 \div 10 = 6$ |
| Fractions | |
| Know unit fraction symbols | $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{10}$ |
| Order fractions with the same denominators | $\frac{1}{4}$, $\frac{3}{4}$ |
| Strategy | |
| I am learning to... | |
| Addition and subtraction Solve simple problems mentally using basic facts I know <ul style="list-style-type: none"> ● Doubles: $8 + 7 = 8 + 8 - 1$ ● Fives: $8 + 7 = 5 + 3 + 5 + 2$ ● Making tens: $8 + 7 = 8 + 2 + 5$ | $8 + 7 =$  <p>Double 8 is 16 so minus 1</p> |
| Addition and subtraction Solve 2 and 3 digit problems by: <ul style="list-style-type: none"> ● Tidy numbers $29 + 18$ as $30 + 17$ ● Place value $33 + 16$ as $30 + 10 + 3 + 6$ |  <p>$29 + 1$ is 30 so now just add 17</p> |
| Multiplication and division Solve problems by: <ul style="list-style-type: none"> ● Using repeated addition with problems involving 2s, 3s, 4s, 5s and 10s at least ● Using doubling additively | $8 \times 5 =$  <p>$5 + 5 = 10$, so $10 + 10 + 10 + 10 = 40$</p> <p>I have 6 baskets of apples, each basket has 15 apples in it. How many apples have I got altogether? $15 + 15 = 30$ $30 + 30 = 60$ $60 + 30 = 90$</p> |

Maths Progressions - Number and algebra

| | |
|---------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | <p>I have 24 lollies. I have to share these between 4 people. How many does each person get?</p> <p>$\frac{1}{2}$ of 24 is 12 and $\frac{1}{2}$ of 12 is 6</p> <p>Or by using trial and improvement with addition</p> |
| <p>Fractions</p> <p>Find a fraction of a number by trial and improvement with addition facts</p> | <p>$\frac{1}{3}$ of 12 =</p>  <p>Which is bigger? $\frac{1}{2}$ of 60 or $\frac{1}{4}$ of 80</p> |
| <p>Fractions</p> <p>Find fractions of shapes and lengths including fractions greater than 1</p> | <p>Ben and his friends ate 16 pieces of cake. Each piece was $\frac{1}{4}$ of the cake. How many cakes did they eat?</p> <p>Examples of strategies for this stage include:</p> <p>Addition strategy: 4 pieces is one cake so $4 + 4 + 4 + 4 = 16$</p> <p>Rate strategies: 4 quarters is one cake so 8 quarters is 2 cakes, so 12 quarters is 3 cakes, so 16 quarters is 4 cakes.</p> <p>Multiplication facts: $4 \times 4 = 16$</p> |
| <p>Equations and Expressions</p> <p>I am learning to...</p> | |
| <p>Communicate and interpret additive strategies using words, numbers, pictures and symbols.</p> |   |
| <p>Pattern and Relationships</p> <p>I am learning to...</p> | |
| <p>Find rules for the next members in a sequential pattern</p> |  |

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| | |
|--------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | <p>How many matchsticks would 10 squares use?</p> <p>20 squares?</p> |
| Identify what the unit of repeat is in a pattern | <p>Identify unit of repeat e.g. what shape goes on the number 14 in this pattern? What colour will it be?</p>  <p>The student identifies the two variables (shape and colour) in the pattern. They might look at the variables separately and identify the unit of repeat for each ("yellow, blue, red" and "triangle, circle").</p> <p>They may look at the variables together and identify the complete unit of repeat ("yellow triangle, blue circle, red triangle, yellow circle, blue triangle, red circle").</p> |

Maths Progressions - Number and algebra

Stage 6 Advanced additive

In year 5 and 6

Year 5-E level 3/Year 6-At level 3

During these school years, number should be the focus of 50-70% of mathematics teaching time.

Mathematical Behaviours (what mathematicians do)

I am learning to...

Explain my mathematical thinking orally, visually, in writing or using digital tools

Select and apply the appropriate representations to solve problems e.g. graphs, diagrams, tables, numbers etc.

Apply the most efficient strategy to solve a problem e.g. I know that for this problem a part-whole place value strategy is more efficient than compensating from tidy numbers.

Engage in mathematical discussions with others

Take or defend a position or point of view about a strategy/answer and justify with evidence e.g recognising relationships, or using counter examples

Explain others' strategies by repeating or re-voicing what they have said

Listen actively

I know where I am going, how I am going and where to next in maths

Use mathematical vocabulary

Make connections with what I am learning in maths to other curriculum areas and daily life

Pose problems and conduct mathematical investigations

Make generalisations

Knowledge

I am learning to...

Sequencing and ordering fractions

Read and order any number up to 1,000,000

Read decimals to 3 decimal places

0.764

"Zero point seven, six, five" OR "seven hundred and sixty four thousandths"

If a child reads this as "zero point seven hundred and sixty four" this is incorrect

Read any fraction including numbers greater than 1

$\frac{8}{6}$ $\frac{4}{5}$ $1\frac{1}{3}$

Order unit fractions

$\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$

Say the number 1, 10, 100 and 1000 more or less

654, 754, 854..

8432, 7432, ...

Count forwards and backwards in $\frac{1}{2}$ s, $\frac{1}{4}$ s, $\frac{1}{3}$ s, $\frac{1}{5}$ s, $\frac{1}{10}$ s

$\frac{8}{10}$, $\frac{9}{10}$, 1, $1\frac{1}{10}$

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| | |
|------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Understand that percentages are out of 100 | At this stage students are not required to convert fractions and decimals. They do not need to rename common fractions as percentages as this comes in at stage 7. However, they should have some understanding that percentages are 'out of 100' and that fractions are percentages - parts of a whole. |
| Make connections between fractions and percentages | |
| Grouping and place value | |
| Know how many tenths, 10s, 100s and 1000s are in whole numbers | 4676 = 467 tens and 46 hundreds 5 = 50 tenths 25 = 250 tenths |
| Know groupings within 1000 | 455 and 555 200 and 800 |
| Know groups of 2s, 3s, 5s and 10s in numbers to 100 and any remainders instantly (using basic facts knowledge) | How many threes in 17? 5 and 2 remainders |
| Round whole numbers to the nearest 10, 100, 1000 | 5508 → 6000 |
| Round decimals to the nearest whole number | 3.49 → 3 |
| Basic facts | |
| Recall all multiplication and division facts to 10 x 10 | 3 x 8 = 24 24 ÷ 5 = 4 7 x 7 = 49 63 ÷ 7 = 9 |
| Recall addition & subtraction facts to 20 | 13 + 5 = 18 16 = 9 + 7 |
| Know what happens when you multiply by 1, 0 or 10 | 14 x 10 = 140 14 x 0 = 0 |
| Strategy | |
| Addition and Subtraction | |
| I am learning to use a broad range of mental strategies to solve addition and subtraction strategies (see strategies below) | |
| I am learning to choose the most suitable strategy for the problem | |
| Compensating from tidy numbers | 394 + 79 (394 + 80) - 1 |

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| | |
|----------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| Place value partitioning | $394 + 79$ $390 + 70 + 9 + 4$ |
| Using compatible numbers | $45 + 37 + 65$ $(45 + 65) + 37$ |
| Using reversibility | $403 = 97 + ?$ $97 + ? = 403$ |
| Using equal additions (add to both numbers) | $403 - 97$ $406 - 100$ |
| Using standard written form for addition and subtraction | $\begin{array}{r} 4394 \\ + 579 \\ \hline \end{array}$ $\begin{array}{r} 2403 \\ - 1097 \\ \hline \end{array}$ |

Multiplication and Division

I am learning to derive multiplication and division facts by (see below)

I am learning to choose the most suitable strategy for the problem

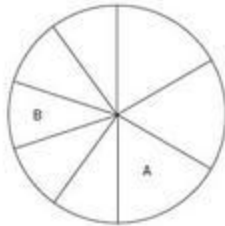

| | |
|----------------------------------------|---------------------------------------------------------------------------------------------------|
| Using doubling and halving | $16 \times 3 \rightarrow 8 \times 3 = 24$ double 24 is 48 $3 \times 12 \rightarrow 6 \times 6$ |
| Using adding and subtracting | $13 \times 3 \rightarrow 12 \times 3 = 36 + 3 = 39$ |
| Using reversing | $63 \div 9 = \rightarrow 9 \times ? = 63$ |
| Using rounding or compensation | $9 \times 6 \rightarrow (10 \times 6) - 6$ |
| Using multiplying by tens and hundreds | $70 \times 5 \rightarrow 7 \times 5 \times 10$ |

Fractions, decimals, ratios and proportions

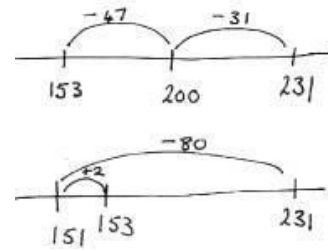
I am learning to solve fraction problems using multiplication and division strategies by...

| | |
|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Finding fractions of whole numbers | $\frac{3}{4}$ of 24 = ? $\frac{3}{4}$ of what is 21? |
| Finding fractions of sets and quantities | <p>On your birthday cake 8 of the lollies are yellow. The other two-thirds of the lollies are red. How many red lollies are on the cake?</p> <p>Possible strategies at this stage:</p> |

Maths Progressions - Number and algebra

| | |
|-----------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | $3 \times 8 = 24$ so 24 lollies in total then adds to get answer $8 + 8 = 16$ Or $3 \times 8 = 24$ so 24 lollies in total - then multiplies to get answer: $2 \times 8 = 16$ Or $1 - \frac{2}{3} = \frac{1}{3}$ $\frac{1}{3} = 8$ $\frac{2}{3} = 8+8$ (or 8×2) = 16 |
| Solving simple equivalent ratio and rate problems | $2 : 3$ so $? : 6$ I add 2 cups of sugar for every 3 cups of flour. How many cups of sugar should I add if I have 6 cups of flour? |
| Solving simple equivalent fraction problems |  <p>Double 3 which is 6. A is in one column so that makes $\frac{1}{6}$.</p> <p>What fractions of the whole birthday cake are pieces A and B? Explain your answer.</p> |
| Comparing fraction sizes with whole numbers | $\frac{37}{7} = 5 \frac{2}{7}$ |
| Equations and Expressions I am learning to... | |
| Record and interpret additive and simple multiplicative strategies using words, diagrams, and symbols with an understanding of equality |  <p>78 plus 22 is 100. 131 more is 231. So the difference between 78 and 231 is 22 plus 131, which is 153.</p> <p>$78 + 22 = 100$ $100 + 131 = 231$ so $231 - 78 = 22$</p> <p>$22 + 131 = 153$</p> |

Maths Progressions - Number and algebra



Pattern and Relationships

I am learning to...

Create and continue sequential patterns with two or more variables

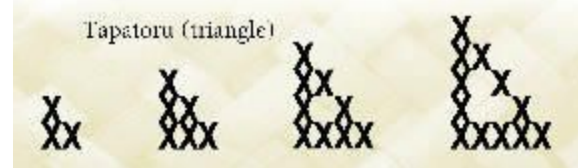
Activity

1. a. i. How many hexagons are in this pattern?
ii. How many green triangles are in this pattern?

b. If you used 16 hexagons to make a pattern like this, how many green triangles would you use?

c. Sonny uses 82 green triangles. How many hexagons will he need to make this pattern?

Predict sequential patterns with two or more variables





This is how the tapatoru pattern grows. How many crosses will be in the 20th tapatoru pattern? Show how you worked out your answer.

The student uses repeated addition or a multiplication rule in conjunction with a recording strategy.



Maths Progressions - Number and algebra

Stage 7 Advanced multiplicative - Early proportional In year 7 and 8 Year 7 - Early level 4/Year 8 - At level 4

During these school years, number should be the focus of 40-60% of mathematics teaching time.

| Mathematical Behaviours (what mathematicians do) | |
|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I am learning to... | |
| Explain my mathematical thinking orally, visually, in writing or using digital tools | Select and apply the appropriate representations to solve problems e.g. graphs, diagrams, tables, numbers etc. |
| Compare a variety of problem solving strategies | Engage in mathematical discussions with others |
| Take or defend a position or point of view about a strategy/answer and justify with evidence e.g recognising relationships, or using counter examples | Apply the most efficient strategy to solve a problem e.g. I know that for this problem a part-whole place value strategy is more efficient than compensating from tidy numbers. |
| Listen actively | I know where I am going, how I am going and where to next in maths |
| Use mathematical vocabulary | Make connections with what I am learning in maths to other curriculum areas, daily life, current events, art, culture or sport |
| Explain others' strategies by repeating or re-voicing what they have said | Pose problems and conduct mathematical investigations |
| Make generalisations | |
| Knowledge | |
| I am learning to... | |
| Sequencing and ordering | |
| Count forwards and backwards in $\frac{1}{1000}$'s, $\frac{1}{100}$'s, $\frac{1}{10}$'s, 1's, 10's, etc including negative numbers. | 1.2, 1.3, 1.4 6.43, 6.43, 6.41 Counting backwards: -24, -25, -26 |
| Say the number $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 10, before or after any number | 1.2, 1.3  , ____ ____  6.42, 6.43 |
| Order decimals to three places | 6.25, 6.3, 6.456 |
| Grouping and place value | |

Maths Progressions - Number and algebra

| | |
|----------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Know groupings of hundredths, tenths, 10s, 100s and 1000s in 7 digit numbers | $3\ 456\ 789 = 345\ 678\ \text{tens}$ $4562 = 620\ \text{tenths}$ |
| Round whole numbers & decimals to nearest 1 or $\frac{1}{10}$ | $0.47 \rightarrow 0.5$ |
| Know groupings of numbers to 10 that are in numbers to 100 and finds the resulting remainders. | How many 6s in 38? 6 remainder 2 |
| Basic facts | |
| Recall all multiplication and division facts to 12×12 | $8 \times 7 = 56$ $72 \div 9 = 8$ |
| Recall conversions between decimals, fractions and percentages with $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{10}$ | $\frac{1}{2}$  0.5  50% |
| Use divisibility rules for 2, 3, 5, 9, 10 | <p>245 is divisible by 5 because the ones column is a 5</p> <p>306 is divisible by 3 and 9 because the sum of the digits ($3 + 0 + 6 = 9$) and 9 is divisible to both 9 and 3</p> |
| Know square numbers and square roots to 100 and corresponding roots | $7^2 = 49$ so $\sqrt{49} = 7$ |
| Identify factors of numbers to 100 including prime numbers | Factors of 35 = 1, 5, 7, 35 |
| Find common multiples of numbers to 10 | Common multiples of 3 and 7 are... 21, 42, 63..... |
| Fractions | |
| Order mixed fractions with $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{10}$ | $\frac{2}{10}, \frac{3}{4}, \frac{1}{2}, \frac{5}{3}$ |
| Know equivalent fractions for $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{5}, \frac{1}{10}$ with denominators 10,100,1000 | $\frac{1}{4} = \frac{25}{100},$ |
| Round whole numbers & decimals to nearest 1 or $\frac{1}{10}$ | <p>0.47 rounded to the nearest tenths is 0.5</p> <p>4.67 rounded to the nearest whole number is 5</p> |
| Recall fraction, decimal to percentage conversions for halves, thirds, quarters, fifths and tenths | <p>$\frac{3}{4} = 0.75 = 75\%$</p> <p>At this stage when learning about percentages students must also understand that like any fraction, there can be percentages greater than 1 e.g. 125%</p> |

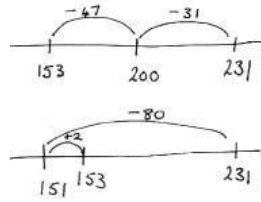
Maths Progressions - Number and algebra

| Strategy | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <p>Addition and subtraction</p> <p>I am learning to solve problems using a broad range of strategies for whole numbers and decimals...</p> <p>I am learning to choose the most suitable strategy for the problem</p> | |
| by compensating from tidy numbers | $3.2 + 1.95$ $(3.2 + 2) - 0.05$ |
| by partitioning using place value | $8.65 + 4.2$ $8 + 4 + 0.6 + 0.2 + 0.05$ |
| by using reversibility | $6.03 - 5.8$ $5.8 + ? = 6.03$ |
| by using equal additions | $7.2 - 3.7$ $7.5 - 4 = 3.5$ |
| by using standard written form (vertical algorithm) | $\begin{array}{r} 7.2 \\ - 3.7 \\ \hline \end{array}$ <p>Student must be able to explain the place value partitioning involved</p> |
| with negatives (integers) | $7 - -3 = 7 + 3 = 10$ |
| balance positive and negative amounts | See this site for examples and models |
| with simple equivalent fractions | $\frac{3}{4} + \frac{3}{8} = \frac{6}{8} + \frac{3}{8} = \frac{9}{8}$ |
| <p>Multiplication and Division</p> <p>I am learning to solve problems using a broad range of strategies by (see below)...</p> <p>I am learning to choose the most suitable strategy for the problem</p> | |
| compensating from tidy numbers | $19 \times 6 = (20 \times 6) - 6$ $56 \div 4 \rightarrow (60 \div 4) - 1$ |
| using place value | $28 \times 7 \rightarrow (20 \times 7) + (8 \times 7)$ $72 \div 4 \rightarrow (40 \div 4) + (32 \div 4)$ |
| using reversibility | $63 \div 9 = 9 \times ? = 63$ |
| using proportional adjustment | $75 \times 4 \rightarrow 25 \times 12$ $81 \div 3 \rightarrow (81 \div 9) \times 3$ |

Maths Progressions - Number and algebra

| | |
|---------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| expressing remainders as fractions, decimals or whole numbers | $38 \div 6 = 6 \text{ r}2$ or $6 \frac{1}{3}$ or 6.33 |
| using standard written forms for \times and \div | 476 $\times 8$ <hr style="width: 10%; margin-left: 0;"/> $6 \overline{)845}$ |
| Fractions, decimals, ratios and proportions I am learning to solve problems by using... | |
| unit fractions | $\frac{5}{8} \times 72 \longrightarrow 5 \times (\frac{1}{8} \times 72)$ |
| place value | $3.4 \times 8 \longrightarrow (3 \times 8) + (0.4 \times 8)$ |
| division | 13 pies to share with 5 people. $13 \div 5 = (10 \div 5) = (3 \div 5) = 2 \frac{3}{5}$ |
| compensation from tidy numbers | $2.9 \times 6.3 =$ $(3 \times 6.3) - (0.1 \times 6.3)$ |
| equivalent fractions and percentages | $40\% \text{ of } 35 = \frac{2}{5} \text{ of } 35$ I got 36/50 goals and Sarah got 16/20. Who was the better shot? $36/50 \times 2 = 72\%$, while $16/20 = 4/5 = 80\%$. Sarah is a better shot. |
| ratios | $3:5$ as :40, $8 \times 5 = 40$, $8 \times 3 = 24$, so = 24 |
| Equations and Expressions I am learning to... | |
| Record the results of calculations using equations and diagrams | $6 \times 28 = 168$ |

Maths Progressions - Number and algebra

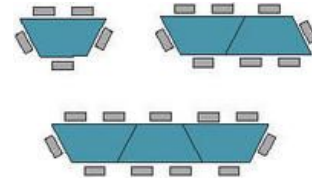


Pattern and Relationships

I am learning to...

Find and represent relationships in spatial and number patterns using

- tables and graphs
- general rules (for linear relationships)



Funky Furniture sells tables that can be joined together for large meetings. Tables and chairs are set up this way.

If a line of 24 tables is set out like this, how many chairs will be needed? Can you give a rule for the number of chairs needed for any given number of tables?

Students would use a table or graph to solve a similar problems to above and identify a general rule.

Find and represent relationships in spatial and number patterns using recursive rule for non-linear relationships (by the end of year 8)

For examples see pages 16 and 22 of FiO book: Algebra Level 4+ year 7-8 (book 4)

Apply inverse operations to simple linear relationships (by the end of year 8)

For a tutorial [click here](#)

Maths Progressions - Number and algebra

Stage 8 Advanced proportional - part whole

In year 9

Level 5

A student in year 8 should have mastered the learning intentions at this stage to be considered 'above'.
A student in year 7 should be comfortably working on these learning intentions at this stage to be considered 'above'.

Mathematical Behaviours (what mathematicians do)

I am learning to...

Explain my mathematical thinking orally, visually, in writing or using digital tools

Select and apply the appropriate representations to solve problems

Compare a variety of problem solving strategies

Engage in mathematical discussions with others

Take or defend a position or point of view about a strategy/answer and justify with evidence e.g recognising relationships, or using counter examples

Apply the most efficient strategy to solve a problem e.g. I know that for this problem a part-whole place value strategy is more efficient than compensating from tidy numbers.

Listen actively

I know where I am going, how I am going and where to next in maths

Use mathematical vocabulary

Make connections with what I am learning in maths to other curriculum areas, daily life, current events, art, culture or sport

Explain others' strategies by repeating or re-voicing what they have said

Pose problems and conduct mathematical investigations

Make generalisations

Knowledge

I am learning to...

Sequencing and ordering

Count forwards and backwards in $\frac{1}{1000}$'s, $\frac{1}{100}$'s, $\frac{1}{10}$'s, 1's, 10's, etc.

1.2, 1.3, 1.4
6.43, 6.43, 6.41

Say the number 0.001, 0.01, 0.1, 1, 10 before and after decimal numbers

6.42, 6.52, _____

Order fractions, decimals and percentages

0.4, 50%, $\frac{4}{5}$

Grouping and place value

Know how many $\frac{1}{10}$'s, $\frac{1}{100}$'s, and $\frac{1}{1000}$'s that are in numbers to 3 decimal places.

1.873 is

Maths Progressions - Number and algebra

| | |
|------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------|
| | 18 tenths, or 187 hundredths etc. |
| Know what happens when any number is multiplied or divided by a power of ten. | $1.23 \times 100 = 123$ $6.53 \div 10 = 0.653$ |
| Round decimals to the nearest 100, 10, 1, 0.1 or 0.01 | 9.876 9.88 |
| Recall fraction, decimal and percentage conversions for commonly used fractions: ($\frac{1}{8}$ s, $\frac{1}{10}$ s, $\frac{1}{20}$ s etc) | $\frac{1}{8} = 0.125 = 12.5\%$ |
| Basic facts | |
| Know simple powers of numbers to 10 | $2^3 = 8$ |
| Use divisibility rules for 2, 3, 4, 5, 6, 8, 9, 10 | 276 is divisible by 3 because $2 + 7 + 6 = 15$ and 15 is divisible by 3 |
| Identify common factors of pairs of numbers to 100 | Highest common factor of 72 & 81 = 9 |
| Identify lowest common multiple of pairs of numbers to 10 | The LCM of 6 & 8 = 24 |
| Recall prime numbers to 20 | e.g. 1, 2, 3, 5, 7... |

| Strategy | |
|----------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| I am learning to solve + - x and \div problems with fractions and decimals by using: | |
| Conversion between fractions and decimals | $0.75 \times 2.4 \longrightarrow$ $\frac{3}{4} \times 2.4$ |
| Place value | $0.15 \times 3.6 \longrightarrow$ $0.1 \times 3.6) + (0.05 \times 3.6)$ |
| Doubling and halving | $7.2 \div 0.4 \longrightarrow$ $(7.2 \div 0.8) \times 2$ |
| Commutativity | $48 \times 0.125 \longrightarrow$ $0.125 \times 8 = \frac{1}{8} \text{ of } 8$ |
| Multiplying numerators and denominators | $\frac{3}{4} \times \frac{2}{5} \longrightarrow \frac{3 \times 2}{4 \times 5}$ |

Maths Progressions - Number and algebra

| | |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Converting to common denominators | $\frac{3}{5} + \frac{2}{7} = \frac{21}{35} + \frac{10}{35} = \frac{31}{35}$ |
| Use written forms for: Addition and subtraction of whole numbers and decimals to 3 decimal places Multiplication and division of whole numbers, decimals and fractions multiplied by a single digit number Multiplication of 4 digit x 2 digit whole numbers | 3.567 $+ 0.063$ <hr/> 6.45 $\times 3$ <hr/> $5 \overline{) 4.83}$ $6 \times 1 \frac{1}{2}$ <hr/> 6735 $\times 85$ <hr/> |
| Find fractions, decimals and percentages of given amounts | <u>Example 1</u> 65% of 24 50% of 24 = 12, 10% of 24 = 2.4 5% of 24 = 1.2 so 65% = 12 + 2.4 + 1.2 <u>Example 2</u> 28 out of 42 = ? % $\frac{28}{42} = \frac{4}{6} = \frac{2}{3} = 66.6\%$ |
| Ratios, rates and proportions Finding equivalent ratios with a common factor or multiplier | $21 : 28$ as $? : 8 \longrightarrow$ $21 : 28 = 3 : 4$ so $6 : 8$ $3 : 5 = ? : ?$ out of 96 \longrightarrow As $3 : 5$ is $\frac{3}{5}$, $\frac{3}{5}$ of 96 = 36, so the proportion is 36 : 60 |
| Equations and Expressions | |
| I can... | |
| Form and solve linear and simple quadratic equations | $5x^2 - 3x + 3 = 0$ <i>this makes it Quadratic</i> |
| Pattern and Relationships | |
| I am learning to... | |
| Generalise the properties of operations with fractional numbers and integers | $-2 + 4 = 4 + (-2)$ $-2(4) = 4(-2)$ Click here for lesson examples. |

Maths Progressions - Number and algebra

Relate tables, graphs and equations to linear and simple quadratic relationships found in number and spatial patterns.

See stage 7 example, however, at this stage students should be able represent linear and simple quadratic patterns in:

- graphs
- tables
- equations

An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 - 3x + 3 = 0$$

Quadratic Equations make nice curves, like this one:

